CALCULATION OF CONTINUUM RADIATIVE ENERGY TRANSFER WITH ALLOWANCE FOR THE VARIATION OF THE ABSORPTION COEFFICIENT WITH FREQUENCY AND REABSORPTION

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 9, No. 1, pp. 98-102, 1968

The method of calculating heat-transfer characteristics proposed in [1,2] for spectral lines is extended to the continuum.

In [1,2] the energy flux and its divergence were calculated with sufficient accuracy by dividing the part of the spectrum important in terms of energy into a small number of intervals and using a specially averaged absorption coefficient on each interval.

In some cases the continuum plays the principal part in radiative energy transfer.

When a plasma can be considered optically thin, the Planck averaging law is applicable. The divergence of the radiant energy flux can be calculated directly. For an optically dense plasma it is usual to employ Rosseland averaging.

However, the radiant energy flux is often formed in the region of intermediate optical density.

The exact expressions for the heat-transfer characteristics in more complicated geometry than a plane layer are very clumsy. Accordingly, we investigated the accuracy of the diffusional approximationf (the first term of the expansion in the spherical-harmonic method) for plane and cylindrical geometrices.

Formulation of the problem. In order to extend the method proposed in [1,2] to the case of a continuous spectrum we examined a m model problem.

The radiative energy transfer (flux and divergence) was calculated for a plane layer of argon 2 cm thick with a given symmetrical temperature profile (Fig. 1).

Local thermodynamic equilibrium was assumed.

The calculations were made for two pressures: p = 1 atm and p = = 100 atm. At 1 atm the radiation associated with free-free transitions and excitation is mainly transmitted and its contribution to heat transfer can be taken directly into account. However, the radiation associated with deexcitation is substantially reabsorbed. Calculations were also made for this process. At 100 atm for the argon layer in question the bremsstrahlung and the radiations associated with transitions to excited levels are reabsorbed. In this case deexcitation makes a negligibly small contribution to the total energy flux.

The absorption coefficient for photoionization from the ground state was calculated from the known photoionization cross section [3].

The absorption coefficient associated with photoionization from excited levels (except transitions from the levels $1s_2 - 1s_5$) and with free-free transitions was calculated from the formulas of [4].

The reduction in ionization potential and the shift of the frequency limit ν_g in the formulas of [4] were taken into account in accordance with the Ecker-Weizel theory.

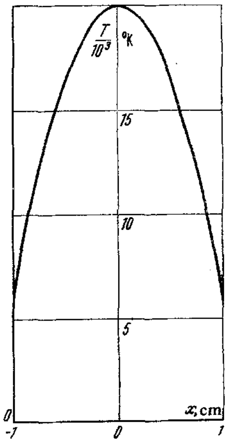

Fig. 1

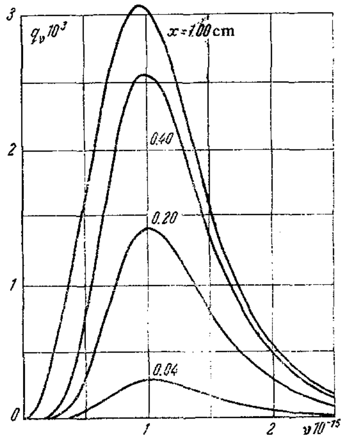

Fig. 2

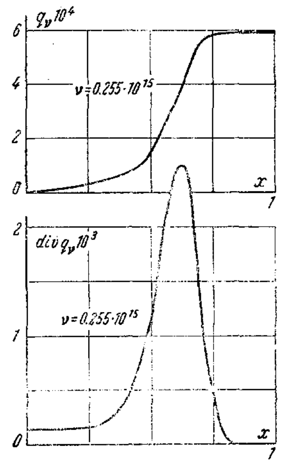

Fig. 3

At frequencies $\nu \geq 0.986 \cdot 10^{15}$ sec^{-1} we also took into account the process of photoionization from the first group of excited levels $1s_2 - 1s_5$. The cross section for this process has been calculated by Norman and is presented in [5].

Calculations were made with the exact expressions for the energy flux and its divergence [6], as well as in the diffusional approximation [6.7]. The error of all the calculations does not exceed 0.5%.

Method of calculating the radiative heat exchange characteristics. The part of the spectrum important in terms of energy is divided into a number of regions.

The energy exchanged in optically transparent spectral regions must be taken directly into account. The contribution of these regions to divergence of the radiant energy flux can be found by integration with respect to the frequency of the energy radiated by a unit volume. It is also possible to use published data on the radiation of hemispherical volumes.

The energy exchanged in optically dense spectral regions can be taken into account in the approximation of radiative heat exchange [6]. Here in evaluating the mean free paths of the photons we must integrate over the given regions.

In the intermediate regions, where the above-mentioned approximations are not applicable, the following method of calculating the heat-transfer characteristics is proposed.

Following [1,2], we divide the intermediate region into intervals on which the absorption coefficient changes approximately the same number of times. This partition should be carried out at the tempera-

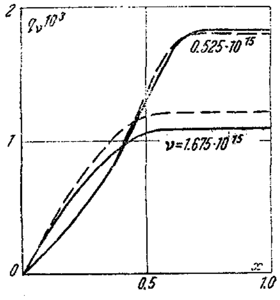

Fig. 4

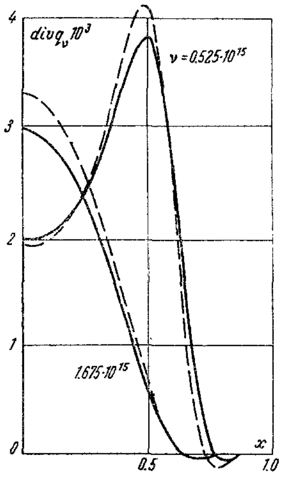

Fig. 5

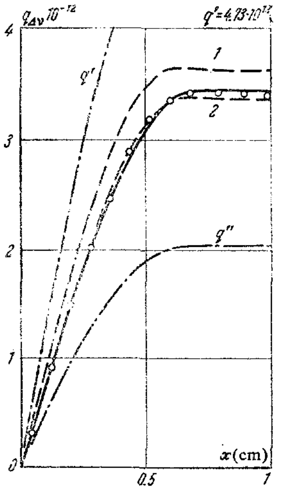

Fig. 6

ture most important for heat transfer in the given problem and the intervals obtained must be fixed. On each of these intervals certain average values of the absorption coefficient are employed.

We introduce the following mean absorption coefficients:

$$k' = \int\limits_{\Delta\nu} k_\nu u_\nu^\circ \, d\nu \left(\int\limits_{\Delta\nu} u_\nu^\circ \, d\nu \right)^{-1}; \tag{1}$$

$$k'' = \int\limits_{\Delta\nu} \frac{du_\nu^\circ}{dT} \, d\nu \left(\int\limits_{\Delta\nu} \frac{1}{k_\nu} \frac{du_\nu^\circ}{dT} \, d\nu \right)^{-1}; \tag{2}$$

$$\langle k \rangle = \sqrt{k' k''}. \tag{3}$$

Here, k_ν is the spectral absorption coefficient and u_ν^0 is the spectral equilibrium density of radiant energy.

Averaging (1) is the analog of Planck averaging and averaging (2) is the analog of Rosseland averaging. These averagings are used in the two extreme cases: transparent (k') and optically dense (k").

As follows from [1, 2], the results of the calculations with k' and k" for the intermediate region deviate from the exact values in essentially different directions. To improve the accuracy we introduced averaging by (3) and employ the coefficients k', k", and $\langle k \rangle$.

Apart from the mean absorption coefficients, we introduce the quantity

$$u_{\Delta\nu}^\circ = \int\limits_{\Delta\nu} u_\nu^\circ \, d\nu$$

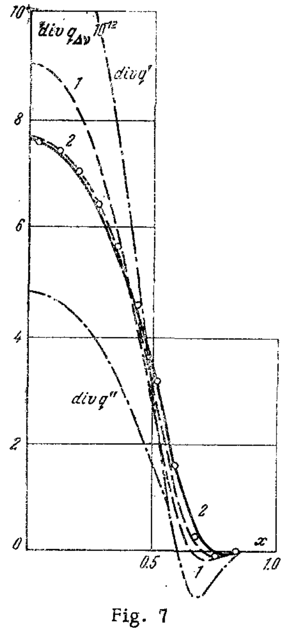

Fig. 7

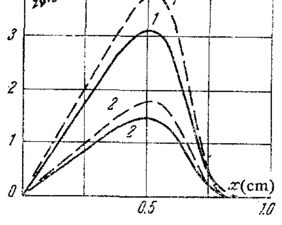

Fig. 8

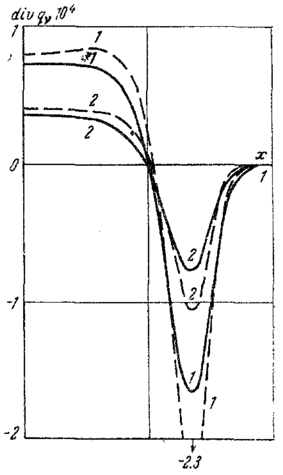

Fig. 9

To calculate the integral radiant energy flux and its divergence by the method proposed we employ expressions analogous to the expressions for the spectral flux and its divergence, in which instead of k_ν we use the mean absorption coefficient, and instead of $\overset{\circ}{u}_\nu$ the quantity $u^\circ{}_{\Delta\nu}$.

Results of the calculations. 1. Pressure p = 100 atm. The relation between the frequency ν ($\sec^{-1}$) and the exact values of the spectral radiant energy flux for different distances from the plane of symmetry is shown in Fig. 2. We note that a considerable fraction of the energy is exchanged in the region where reabsorption is important. In Figs. 3, 4, and 5 the solid lines show the variation of q_ν erg/cm$^2\cdot$sec$\cdot$sec^{-1} and div q_ν erg/cm$^3\cdot$sec$\cdot$sec^{-1} across the layer (the value of x is reckoned from the plane of symmetry) for several frequencies. These figures show the characteristic behavior of the quantities q_ν and div q_ν for different optical densities. Figure 3, in particular, corresponds to a high optical density.

To calculate the quantities $q_{\Delta\nu}$ and div $q_{\Delta\nu}$, integral in frequency, and to check the approximate method of calculation we selected the frequency region $\Delta\nu$ from $\nu = 0.27 \cdot 10^{15}$ sec^{-1} to $\nu = 2.75 \cdot 10^{15}$ sec^{-1}. The contribution to the total radiant energy flux and its divergence from the spectral regions outside this interval is less than 4%. Inside the selected frequency region the absorption coefficient k_ν changes by a factor of approximately 250.

The exact integral values of $q_{\Delta\nu}$ and div $q_{\Delta\nu}$ are represented by the solid lines in Figs. 6 and 7.

With the mean absorption coefficients k', k", and $\langle k \rangle$ we calculated $q_{\Delta\nu}$ and div $q_{\Delta\nu}$ by averaging over the entire region $\Delta\nu$ and by dividing that region into two and three intervals as indicated above. The results of the calculations are shown in Figs. 6 and 7. The dot-

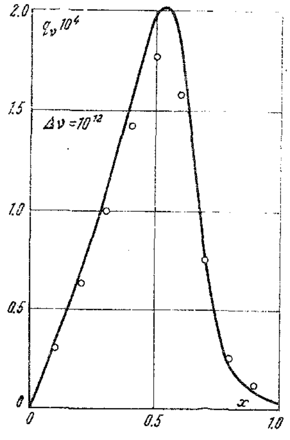

Fig. 10

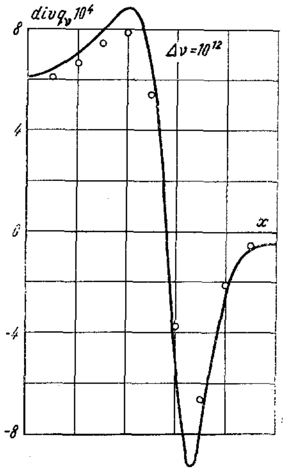

Fig. 11

dash lines represent the values obtained with the coefficients k' and k" for the entire region $\Delta\nu$. It is clear that the deviation from the exact values is considerable. The dashed line represents the values obtained with $\langle k \rangle$. The figures on the dashed-line curves denote the number of intervals into which the region $\Delta\nu$ was divided. The circles represent values obtained by dividing the region $\Delta\nu$ into three intervals.

Clearly, even for the coefficient $\langle k \rangle$ averaged over the entire region $\Delta\nu$ the deviation from the exact solution is not great. When the region $\Delta\nu$ is divided into intervals the error decreases rapidly.

2. Pressure p = 1 atm. The solid lines in Figs. 8 and 9 represent the variation of the spectral flux and its divergence across the layer for two values of the frequency (curves 1 for $\nu = 3.95 \cdot 10^{15}$ sec^{-1}, and curves 2 for $\nu = 4.35 \cdot 10^{15}$ sec^{-1}).

The region of integration was taken from the threshold value of $\nu = 3.811 \cdot 10^{15}$ to $\nu = 5.55 \cdot 10^{15}$ sec^{-1}. About 4% of the energy is transported at $\nu > 5.55 \cdot 10^{15}$ sec^{-1}.

Since the absorption coefficient varies only slightly with respect to frequency in the region of integration (by 25%, while across the layer it changes by a factor of about 400), the use of the coefficient averaged over the entire region $\Delta\nu$, led almost to coincidence with the exact results.

3. Diffusional approximation. In [1,2] the diffusional approximation for a plane layer was compared with the exact solution in the case of spectral lines. For a continuous spectrum, the results obtained in this paper for the spectral quantities q_ν and $\mathrm{div}\, q_\nu$ in the diffusional approximation for a plane geometry are represented by the dashed lines in Figs. 4, 5, 8, and 9. From a comparison with the corresponding exact quantities (solid lines) it is clear that the accuracy of the diffusional approximation is the same as in the case of spectral lines. A similar comparison was made for the case of a cylindrical geometry. Radiative energy transfer in the argon resonance line $\nu_0 = 2.86 \cdot 10^{15}$ sec^{-1} was considered at a distance from its center of $\nu - \nu_0 = 10^{12}$ sec^{-1} (see [1, 2]). The calculations were made for a cylindrical column of argon at p = 1 atm with the temperature distribution shown in Fig. 1.

The exact values of q_ν and $\mathrm{div}\, q_\nu$ were calculated by means of integral relations and are shown in Figs. 10 and 11 as small circles. Values of the same quantities in the diffusional approximation were obtained from a numerical solution of the differential equations by V. N. Vetlutskii and N. V. Zarovna.

The results of their calculations are represented by the solid lines in Figs. 10 and 11. A comparison of Figs. 10 and 11 with the analogous figures of [2] shows that in the case of a cylindrical geometry the diffusional approximation gives approximately the same accuracy as a plane geometry.

Thus, the diffusional approximation can be employed in calculations that do not require very great accuracy.

The authors thank A. T. Onufriev.

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18 March 1967 Novosibirsk